Complex Spatial Dynamics
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Abstract.
The work done with this grant was on the topic of complex spatial dynamics, and included progress in three areas: (i) analysis of the space of cellular automaton rules, particularly with an eye toward the question of where complex cellular aoutomata live in this space, and (ii) analyzing data from complex spatial systems, and (iii) nonlinear resonance and control.

1. Complex Cellular Automata

S. Wolfram, in his studies of cellular automata, observed four broad classes of asymptotic behavior of a cellular automaton rule starting from a random initial condition [1]. This crude classification scheme, while capturing the general idea that there are different types of behavior, suffered from several problems. The biggest problem was that the classification was ambiguous because now quantitative method was given for distinguishing between classes. This problem was particularly troublesome for Wolfram's "class four" rules, which were the rules that generate complex patterns in space-time. Another problem was that Wolfram's classification gave no inkling of how rules in different classes were related.

Li and Packard began to address the cellular automata classification problem first restricting attention to elementary cellular automata (those with nearest neighbors and two-states per site) [2]. They mapped out the space of elementary rules, using a Hamming distance metric on the space of rules to map out the connectivity properties, and a statistical criterion for chaos based on spreading rates. The general picture, with a few exceptions, was that simple rules were connected, chaotic rules were connected, and rules with long complex transients came in between.

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Li, Langton and Packard continued the analysis of more general spaces of cellular automaton rules [3]. They found that the space of rules is generally composed of rules with simple asymptotic behavior and rules with chaotic behavior, with complex behavior sandwiched in between. Paths from simple rules to chaotic rules, obtained by changing a rule table by one Hamming unit along the path, usually run through rules that have complex asymptotic dynamics. Complexity of cellular automaton rules was characterized by perturbation spreading rates, block entropies, and convergence rates of these quantities.

2. Analyzing Data from Complex Spatial Systems

The main idea behind this work has been to use *learning algorithms* to build models directly from complex data. This approach has been implemented in a variety of ways for low dimensional chaotic systems; the aim of the research reported here has been to attack the problem of building models for spatial systems.

The research began with the goal of reconstructing dynamical laws from complex space-time data. The tools have since been considerably generalized to treat a wider range of complex data in more general contexts (not just from spatio-temporal systems). The basic idea of all the algorithms is to search for good dynamical models in a very large possible space of possible models. In different contexts, the search is accomplished with different versions of the genetic algorithm that have genetic codes for different types of models.

Our research in this area began with the analysis of data from a dendritic crystal growth experiment (using $BrNH_4$ crystals growing in a supercooled solution). An intermediate goal was the analysis of artificial data produced by cellular automaton rules that had the property of dendritic growth [4]. We then proceeded to analyze the crystal growth data directly [5].

The learning algorithm used in the analysis was a version of the genetic algorithm that was exploring a space of genetically encoded models. The models were assigned a "fitness" based on the degree to which the complex growth could be reproduced by the learned model.

For analyzing the crystal growth data, the space of models searched by the genetic algorithm was a space of probabilistic cellular automaton rules. The local neighborhood of the cellular automaton was genetically encoded, and the transition probabilities were filled directly from observations of sequences of images taken from the actual growing crystal.

There are two directions that this research is currently aimed. One direction is still connected with spatial systems; the goal is to make as sharp predictions of special space-time features as possible, instead of building a model with the best overall predictability. This research is ultimately aimed at predicting storms from sequences of weather satellite images, for example.

The second direction is the generalization of the genetic algorithm to explore other types of models, for other contexts besides complex spatiotemporal data [6]. This approach has had some striking success in building predictive models for low and high dimensional chaos, as well as finding dynamical patterns in a variety of other places from EEG data to pseudorandom number generators.

3. Nonlinear Resonance and Control

The basic idea of this research has been to use a model for the nonlinear dynamics of an observed system to create a maximally efficacious, or optimal driving force.

In one case, a method of probing a high dimensional system with aperiodic driving forces in order to ascertain its nonlinear structure was designed [7]. Another application of these ideas was to design optimal driving forces that could reduce the complexity of the dynamics of the system, for example, to force the dynamics from a high dimensional strange attractor to a low dimensional fixed point or limit cycle attractor.

In resonance applications, a system may be stimulated extremely efficiently by using knowledge of the system's equations of motion. In typical nonlinear systems, excitation using sinusoidal driving forces, as is most common in spectroscopic applications, will not result in efficient stimulation of the system. If, instead, the nonlinear system is excited with an aperiodic force designed to match its dynamics, extremely efficient stimulation is achieved [9]

References

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